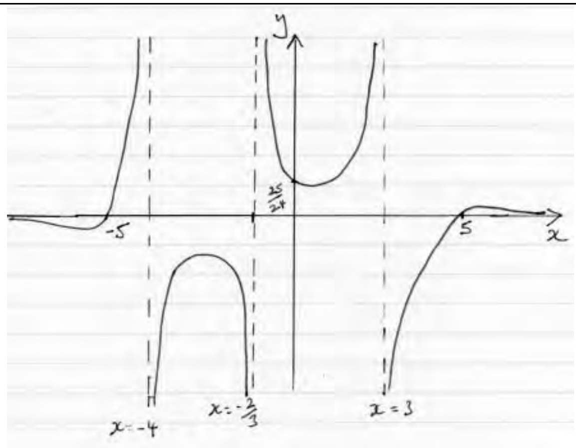
Question			Answer	Marks	Guidance
1	(i)		Transformation A is a reflection in the y-axis. Transformation B is a rotation through 90° clockwise about the origin.	B1 B1 **[2]**	
1	(ii)		$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply in correct order cao
1	(iii)		Reflection in the line $y = x$	B1 **[1]**	
2	(i)		$\lvert z_1 \rvert = \sqrt{3^2 + \left(3\sqrt{3}\right)^2} = 6$ $\arg\left(z_1\right) = \arctan\frac{3\sqrt{3}}{3} = \frac{\pi}{3}$	M1 A1 M1 A1 **[4]**	Use of Pythagoras cao cao
2	(ii)		$z_2 = \frac{5}{2} + \frac{5\sqrt{3}}{2}\text{j}$	M1 A1 **[2]**	May be implied cao
2	(iii)		Because z_1 and z_2 have the same argument	E1 **[1]**	Consistent with (i)
3			$\alpha + \frac{\alpha}{6} + \alpha - 7 = \frac{-8}{3} \Rightarrow \alpha = 2$ Other roots are -5 and $\frac{1}{3}$ Product of roots = $\frac{-q}{3} = \frac{-10}{3} \Rightarrow q = 10$ Sum of products in pairs $= \frac{p}{3} = -11 \Rightarrow p = -33$	M1 A1 M1 A1 M1 A1	Attempt to use sum of roots Value of α (cao) Attempt to use product of roots $q = 10$ c.a.o. Attempt to use sum of products of roots in pairs $p = -33$ cao

Question			Answer	Marks	Guidance
			OR, for final four marks		
			$(x-2)(x+5)(3x-1)$	M1	Express as product of factors
			$=3x^3+8x^2-33x+10$	M1	Expanding
			$\Rightarrow p=-33$ and $q=10$	A1	$p=-33$ cao
				A1	$q=10$ cao
				[6]	
4			$\frac{3}{x-4}>1 \Rightarrow 3(x-4)>(x-4)^2$	M1*	Multiply through by $(x-4)^2$
			$\Rightarrow 0>x^2-11x+28$		
			$\Rightarrow 0>(x-4)(x-7)$	M1dep*	Factorise quadratic
			$\Rightarrow 4<x<7$	B2	One each for $4<x$ and $x<7$
			OR		
			$\frac{3}{x-4}-1>0 \Rightarrow \frac{7-x}{x-4}>0$	M1*	Obtain single fraction >0
			Consideration of graph sketch or table of values/signs	M1dep*	
			$\Rightarrow 4<x<7$	B2	One each for $4<x$ and $x<7$
			OR		
			$3=x-4 \Rightarrow x=7$ (each side equal)		
			$x=4$ (asymptote)		
			Critical values at $x=7$ and $x=4$	M1*	Identification of critical values at $x=7$ and $x=4$
			Consideration of graph sketch or table of values/signs	M1dep*	
			$4<x<7$	B2	One each for $4<x$ and $x<7$
			OR		
			Consider inequalities arising from both $x<4$ and $x>4$	M1*	
			Solving appropriate inequalities to their $x>7$ and $x<7$	M1dep*	
			$4<x<7$	B2	One for each $4<x$ and $x<7$, and no other solutions
				[4]	

Question			Answer	Marks	Guidance
5	(i)		$\frac{1}{2r+1}-\frac{1}{2r+3}=\frac{2r+3-(2r+1)}{(2r+1)(2r+3)}=\frac{2}{(2r+1)(2r+3)}$	M1 A1 **[2]**	Attempt at common denominator
5	(ii)		$\sum_{r=1}^{30}\frac{1}{(2r+1)(2r+3)}=\frac{1}{2}\sum_{r=1}^{30}\left[\frac{1}{2r+1}-\frac{1}{2r+3}\right]$ $=\frac{1}{2}\left[\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{59}-\frac{1}{61}\right)+\left(\frac{1}{61}-\frac{1}{63}\right)\right]$ $=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{63}\right)=\frac{10}{63}$	M1 M1 M1 A1 A1 **[5]**	Use of (i); do not penalise missing factor of $\frac{1}{2}$ Sufficient terms to show pattern Cancelling terms Factor ½ used oe cao
6	(i)		$a_2=3\times2=6, a_3=3\times7=21$	B1 **[1]**	cao
6	(ii)		When n = 1, $\frac{5\times3^0-3}{2}=1$, so true for n = 1 Assume $a_k=\frac{5\times3^{k-1}-3}{2}$ $\Rightarrow a_{k+1}=3\left(\frac{5\times3^{k-1}-3}{2}+1\right)$ $=\frac{5\times3^k-9}{2}+3=\frac{5\times3^k-9+6}{2}$ $=\frac{5\times3^k-3}{2}=\frac{5\times3^{(k+1)-1}-3}{2}$ But this is the given result with k + 1 replacing k. Therefore if it is true for $n=k$ it is also true for $n=k+1$. Since it is true for n = 1, it is true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	Showing use of $a_n=\frac{5\times3^{n-1}-3}{2}$ Assuming true for $n=k$ a_{k+1}, using a_k and attempting to simplify Correct simplification to left hand expression. May be identified with a 'target' expression using $n=k+1$ Dependent on A1 and previous E1 Dependent on B1 and previous E1

Question			Answer	Marks	Guidance
7	(i)		$(-5,\ 0),\ (5,\ 0),\left(0,\ \frac{25}{24}\right)$	B1 B1 B1 **[3]**	-1 for each additional point
7	(ii)		$x=3$, $x=-4$, $x=-\frac{2}{3}$ and $y=0$	B1 B1 B1 B1 **[4]**	
7	(iii)		Some evidence of method needed e.g. substitute in 'large' values or argument involving signs Large positive x, $y \to 0^+$ Large negative x, $y \to 0^-$	M1 B1 B1 **[3]**	
7	(iv)			B1* B1dep* B1 B1 **[4]**	4 branches correct Asymptotic approaches clearly shown Vertical asymptotes correct and labelled Intercepts correct and labelled

Question			Answer	Marks	Guidance
8	(i)		$3(1+3\text{j})^3-2(1+3\text{j})^2+22(1+3\text{j})+40$ $=3(-26-18\text{j})-2(-8+6\text{j})+22(1+3\text{j})+40$ $=(-78+16+22+40)+(-54-12+66)\text{j}$ $=0$ So $z=1+3j$ is a root	M1 A1 A1 A1 **[4]**	Substitute $z=1+3\text{j}$ into cubic $(1+3\text{j})^2=-8+6\text{j}$, $(1+3\text{j})^3=-26-18\text{j}$ Simplification (correct) to show that this comes to 0 and so $z=1+3\text{j}$ is a root
8	(ii)		All cubics have 3 roots. As the coefficients are real, the complex conjugate is also a root. This leaves the third root, which must therefore be real.	E1 **[1]**	Convincing explanation
8	(iii)		$1-3\text{j}$ must also be a root Sum of roots = $-\frac{-2}{3}=\frac{2}{3}$ **OR** product of roots $=-\frac{40}{3}$ **OR** $\sum\alpha\beta=\frac{22}{3}$ $(1+3\text{j})+(1-3\text{j})+\alpha=\frac{2}{3}$ **OR** $(1+3j)(1-3j)\alpha=-\frac{40}{3}$ **OR** $(1-3j)(1+3j)+(1-3j)\alpha+(1+3j)\alpha=\frac{22}{3}$ $\Rightarrow \alpha=\frac{-4}{3}$ is the real root	B1 M1 A2,1,0 A1	 Attempt to use one of $\sum\alpha, \alpha\beta\gamma, \sum\alpha\beta$ Correct equation Cao
			OR $1-3\text{j}$ must also be a root $(z-1+3\text{j})(z-1-3\text{j})=z^2-2z+10$ $3z^3-2z^2+22z+40\equiv(z^2-2z+10)(3z+4)=0$ $\Rightarrow z=\frac{-4}{3}$ is the real root	 B1 M1 A1 A1 A1 **[5]**	 Use of factors Correct quadratic factor Correct linear factor (by inspection or division) Cao

Question			Answer	Marks	Guidance
9	(i)		$p=7\times(-4)+(-1)\times(-19)+(-1)\times(-9)=0$	E1	AG must see correct working
			$q=2\times11+1\times(-7)+k\times(2-k)$	M1	
			$\Rightarrow q=15+2k-k^2$	A1	AG Correct working
				[3]	
9	(ii)		$\mathbf{AB}=\begin{pmatrix}79&0&0\\0&79&0\\0&0&79\end{pmatrix}$	B2	-1 each error
			$\mathbf{A}^{-1}=\frac{1}{79}\begin{pmatrix}-4&-5&11\\-19&-4&-7\\-9&-31&5\end{pmatrix}$	M1	Use of **B**
				B1	$\frac{1}{79}$
				A1	Correct inverse
				[5]	
9	(iii)		$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{79}\begin{pmatrix}-4&-5&11\\-19&-4&-7\\-9&-31&5\end{pmatrix}\begin{pmatrix}14\\-23\\9\end{pmatrix}=\begin{pmatrix}2\\-3\\8\end{pmatrix}$	M1	Attempt to pre-multiply by their $\mathbf{A}^{-1}$
			$\Rightarrow x=2,\ y=-3,\ z=8$	A1	SC A2 for x, y, z unspecified
				A1	sSC B1 for A^{-1} not used or incorrectly placed.
				A1	
				[4]	